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LETTER TO THE EDITOR

Operator content of the Blume–Capel quantum chain

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Abstract. We calculated the conformal anomaly number of the Blume-Capel quantum chain via finite-size corrections to the ground-state energy. In addition we compute, at the tricritical point, the lowest excitations of the spectrum to be compared with the theoretical description in terms of the irreducible representations of the corresponding Virasoro algebra.

Recently it was noticed by several authors (Belavin *et al* 1984, Dotsenko 1984, Friedan *et al* 1984, Cardy 1984) that, in two dimensions, conformal invariance severely restricts the behaviour of the correlation functions. At the same time, Cardy (1984a, b) has shown how to combine this invariance and finite-size scaling to obtain bulk quantities from the transfer matrix ($T=\exp(-\tau H)$) of the finite-width infinitely long strip. The above method was successfully applied in studying a large set of challenging problems such as the eight-vertex and q-state Potts models (Nightingale and Blöte 1983). Subsequently the method has proved its utility in the analysis of quantum Hamiltonians associated with anisotropic models (Alcaraz and Drugowich de Felício 1984, Penson and Kolb 1984, Gehlen *et al* 1984, Alcaraz *et al* 1985) as well in investigating the non-universal behaviour of inhomogeneous models (Turban 1985, Guimarães and Drugowich de Felício 1986).

However, the unusual coupling of the relevant parameter $1/L$ to the universal quantities of the model kept one more fundamental relation hidden. As shown by Blöte *et al* (1986) and Affleck (1986) the finite-size corrections to the lowest eigenvalue E_0 of the Hamiltonian H are closely related to the conformal anomaly number c, which describes the particular realisation of conformal symmetry in the theory. In the case of periodic boundary conditions this remarkable relation is written

$$E_0^L = fL - \pi\gamma c/6L \tag{1}$$

where f is the thermodynamic limit of the ground-state energy per site and γ is a constant which restores the conformal invariance lost in the anisotropic limit. Equation (1) becomes more interesting when the number c is less than one since in this case the model must fall (if the number of primary operators is finite) into the classification of Friedan *et al* (1984). Therefore we can use the BPZ parametrisation

$$c = 1 - 6/m(m+1) \tag{2}$$

and the Kac formula

$$h_{p,q} = \frac{[p(m+1)-qm]^2-1}{4m(m+1)} \qquad 1 \leqslant q \leqslant p \leqslant m-1 \tag{3}$$

to obtain all the scaling dimensions ($x = h + \bar{h}$) of the primary operators. In this sense equation (1) was used by Gehlen and Rittenberg (1986) to study the three-state Potts

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quantum chain. The result $c = 0.800\,08(1)$ is in complete agreement with the expected value $c = \frac{4}{5}$ associated with $m = 5$. Knowing m, equation (3) gave the complete set of irreducible representations $(h, \bar{h})$ of the corresponding Virasoro algebra, which in turn was used to understand the spectrum of H.

Motivated by the success of the above investigation we decided to study the Blume-Capel quantum chain (Blume 1966, Capel 1966) in order to extract its conformal anomaly number and to understand its spectrum at the tricritical point. The renewed interest in this model is twofold: first, it is not an exactly solvable model and, second, because at the tricritical point (see figure 1) it is expected to exhibit supersymmetry (a generalisation of conformal symmetry which includes symmetry between bosonic and fermionic fields (Qiu 1986)). The Hamiltonain we have studied is

$$\hat{H} = -\sum_i \{S_z(i)S_z(i+1) - \alpha S_z^2(i) - \beta S_x(i)\} \tag{4}$$

where $S_x(i)$, $S_z(i)$ are quantum spin-one operators represented by

$$S_\omega(i) = \mathbb{1} \otimes \mathbb{1} \otimes \cdots \otimes \hat{S}_\omega \otimes \cdots \otimes \mathbb{1}$$

with

$$\hat{S}_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \hat{S}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{5}$$

acting on the product Hilbert space. This Hamiltonian commutes with the operator

$$\mathcal{P} = \pi \bigotimes_i [2\hat{S}_x^2(i) - \mathbb{1}] \tag{6}$$

which allows us to divide the spectrum into two sectors (even and odd). In order to calculate the central charge c of this model we need to obtain, for several lattices, the

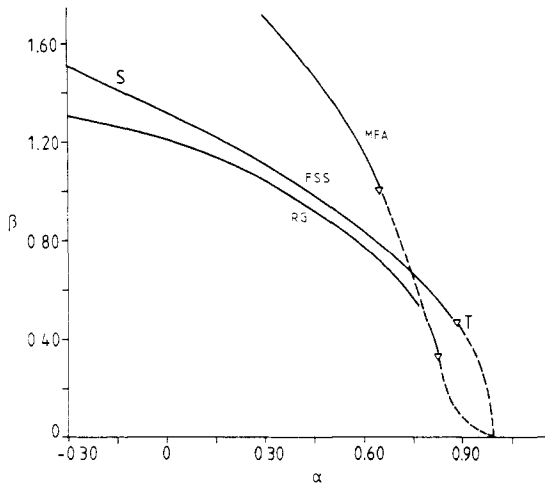

Figure 1. Phase diagram of the Blume-Capel Hamiltonian (4). The mean-field approximation (MFA) and the real space renormalisation group (RG) calculations were done by Hamber (1980) (see also Gefen *et al* 1981, Boyanovsky and Masperi 1980), whereas the finite-size study (FSS) was carried out by Alcaraz *et al* (1985). The triangle (∇) locates the tricritical point T($\alpha = 0.9103$, $\beta = 0.4155$) separating the first-(broken) and second-(full) order transition lines.

ground-state energy per site and to extrapolate it to the thermodynamic limit ($L \to \infty$). Next we obtain the factor γ by calculating the gap which separates the lowest energies of the odd sector, with momentum (k) equal to zero and $2\pi/L$ (this gap should be equal to $2\pi\gamma/L$).

In table 1 we present the estimates of the ratio (E_0/L) at the points T (tricritical) and S (critical) (see figure 1). In tables 2 and 3 we list, respectively, estimates of the

Table 1. Ground-state energy per site (E_0/L) of the Blume-Capel quantum chain at the point T (first column) and S (second column).

L	$-E_0/L$ at point T	$-E_0/L$ at point S
2	0.233 778 43	1.908 612 48
3	0.201 751 77	1.857 373 02
4	0.191 145 37	1.840 789 77
5	0.186 338 38	1.833 355 18
6	0.183 754 09	1.829 381 24
7	0.182 204 98	1.827 007 15
8	0.181 203 24	1.825 475 17
9	0.180 518 12	1.824 428 89
Extr.	0.177 9512(2)	1.820 520(5)

Table 2. Sequence of estimates for the factor γ (see (1)) at the points T (first column) and S (second column).

L	T	S
2	0.373 025	0.689 156
3	0.478 083	0.932 940
4	0.517 130	1.038 314
5	0.535 363	1.092 954
6	0.545 205	1.124 687
7	0.551 066	1.144 629
8	0.554 811	1.157 925
9	0.557 290	1.167 207

Table 3. Sequence of estimates for the conformal anomaly number (c) of the Blume-Capel quantum chain.

L	c at point T	c at point S
2	1.143 451	0.976.517
3	0.855 933	0.678 981
4	0.780 001	0.596 528
5	0.748 558	0.560 692
6	0.732 563	0.541 680
7	0.723 441	0.530 339
8	0.717 773	0.523 017
9	0.712 548	0.518 008
Extr.	0.70(2)	0.49(7)

factor γ and of the conformal anomaly number in both cases. The extrapolated values of table 3 are in complete agreement with the expected values $c=\frac{7}{10}$ $(m=4)$ at the tricritical point and $c=\frac{1}{2}$ $(m=3)$ at the Ising-like critical line.

In proceeding further, we concentrate our attention on the tricritical point where the whole spectrum must be described by the irreducible representations of the Virasoro algebra associated with $c=\frac{7}{10}$, whose dimensions are

$$0$$
$$\frac{7}{16} \quad \frac{3}{80}$$
$$\frac{1}{10} \quad \frac{3}{5} \quad \frac{3}{2}.$$

In order to check the above statement we calculated the scaled gaps:

$$\mathscr{E}_i(P)=\lim_{L\to\infty}\frac{L}{2\pi\gamma}\left(E_i^L(P)-E_0^L(0)\right) \tag{7}$$

where $E_i^L(P)$ is the energy of the ith excited state of H whose momentum is $k=2\pi P/L$. The experimental results obtained (see table 4) should be compared with the theoretical predictions of the conformal invariance (Cardy 1984a, Gehlen and Rittenberg 1986), namely

$$\mathscr{E}_i(P)=(h+r)+(\bar{h}+\bar{r}) \qquad r=0,1,2,\ldots \tag{8a}$$

with

$$P=(h+r)-(\bar{h}+\bar{r}). \tag{8b}$$

Because of the periodic boundary conditions only operators with integer spin may occur, which means in this case $h=\bar{h}$. So the number of irreducible representations describing the spectrum of the tricritical Blume-Capel (periodic) quantum chain is six: $(0,0)$, $(\frac{3}{80},\frac{3}{80})$, $(\frac{7}{16},\frac{7}{16})$, $(\frac{1}{10},\frac{1}{10})$, $(\frac{3}{5},\frac{3}{5})$ and $(\frac{3}{2},\frac{3}{2})$.

To conclude we mention that in the case of free boundary conditions the spectum of H is described only by two representations (0 and $\frac{3}{2}$) and the anomalous dimension of the surface magnetisation equals $\frac{3}{2}$, as we show in table 5. These results confirm a

Table 4. The experimental spectrum $\mathscr{E}_i(P)$ together with its theoretical interpretation (8(a) and (b)).

P	$h+r+\bar{h}+\bar{r}$	$(0,0)$	$(\frac{3}{80},\frac{3}{80})$	$(\frac{7}{16},\frac{7}{16})$	$(\frac{1}{10},\frac{1}{10})$	$(\frac{3}{5},\frac{3}{5})$	$(\frac{3}{2},\frac{3}{2})$	$\mathscr{E}_i^{(+)}(P)$	$\mathscr{E}_i^{(-)}(P)$
0	0.075	—	1	—	—	—	—		0.075(1)
	0.2	—	—	—	1	—	—	0.199(1)	
	0.875	—	—	1	—	—	—		0.87(6)
	1.2	—	—	—	—	1	—	1.18(7)	
	2.075	—	1	—	—	—	—		2.07(7)
	2.2	—	—	—	1	—	—	2.19(2)	
	2.875	—	—	1	—	—	—		2.8(5)
	3.0	—	—	—	—	—	1	3.0(6)	
1	1.075	—	1	—	—	—	—		1.075(6)
	1.2	—	—	—	1	—	—	1.1(9)	
	1.85	—	—	1	—	—	—		1.87(8)
2	2.0	1	—	—	—	—	—	2.0	
	2.075	—	1	—	—	—	—		2.0(7)

Table 5. Estimates for the anomalous dimension of the surface magnetisation $X_s = 2\mathscr{E}_1^{(-)}/\mathscr{E}_1^{(+)}$.

L	X_s
2	1.624 915
3	1.622 975
4	1.596 981
5	1.579 588
6	1.567 248
7	1.558 093
8	1.551 061
9	1.545 508
Extr.	1.50(2)

recent proposal by Cardy (1986b) based on the Landau-Ginzburg-Wilson formulation of the model.

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